Technical Note
Twin Rotor Hover Performance

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During 1947, Mr. Wieslaw Z. Stepniewski conducted a milestone model test that determined the influence on hover performance when co-planar, twin rotors are overlapped. The original thrust and power data was, however, given very limited distribution in 1948. This data, in tabulated form, has been recovered from the archives and included in this technical note. The experimental results provide a valid data base for future theory versus test correlation. In addition, the "psuedo ideal" performance of co-planar, twin rotors is given using blade element-momentum theory and assuming constant chord blades twisted as $\theta_{(x)} = \theta_{tip}/x$. An engineering approximation to rapidly calculate co-planar, twin rotor hover performance for any overlap ratio and at practical rotor loading is offered.

Introduction

In 1947, Mr. Wieslaw Z. Stepniewski and a small team of enthusiasts, working at Piasecki Helicopter Corporation, conducted a fundamental tandem rotor hover performance test. The experiment measured thrust and power required by two, 3-bladed, 4-foot diameter rotors. Hover out of ground effect performance was obtained at four overlapped, co-planar positions. Funding for this test came as part of the U. S. Navy program to develop the XHJP-1, which became the HUP series when this nearly 6,000 pound gross weight tandem helicopter went into production. The model thrust and power data, though published in Refs. 1 and 2, received very limited distribution. However, Mr. Stepniewski and his team semi-empirically reduced the thrust and power data to a simple overlap correction factor that was designated K_{ov}. This factor accounted for increasing power when two rotors are overlapped while holding total thrust constant. In its original form given in Ref. 2, the correction factor applied to the total power. Later, by subtracting a roughly approximated profile power from the total power, the K_{ov} factor became a correction to just induced power. This semi-empirical step was described in detail in Ref. 3 although the results had been summarized earlier in Ref. 4. Reference 5 is a modern source discussing this K_{ov} factor as well as thrust and power of intermeshing and overlapping rotors.

The primary purpose of this Technical Note is to make available to current and future researchers, the original thrust and power measurements upon which the modern day K_{ov} factor is based. Secondarily, a simple theory is provided that confirms the experimental trends Mr. Stepniewski and his small team observed in their 1947 model test.

Stepniewski 1947 Overlap Test

The procedure Mr. Stepniewski used during his co-planar, tandem rotor overlap experiment was quite straight forward. The fore and aft, four foot diameter rotors were first individually tested to obtain baseline isolated rotor performance. Then both rotors were set to a nominally equal collective pitch angle. Finally, the two rotors were "slid together" starting from a non-overlapped position of d/D = 1.0365. Repositioning for each overlap required stopping the model and hand adjusting the movable rear rotor towards the front rotor. Data was recorded at successive overlaps of d/D = 1.0365, 0.8802, 0.7604, and 0.6250. This overlap sweep was repeated six times. The lowest collective pitch of 7 degrees was tested at 1570 and 1780 RPM and the mid collective of 9 degrees at 1570, 1780 and 2015 RPM. The highest collective pitch, 11.5 degrees was tested only at 1570 RPM. The RPM range gives tip speeds of 330, 383 and 422 feet per second. The tip Reynolds number ranges from 264,000 to 337,000.

The primary test results from Mr. Stepniewski's original report (Refs. 1 and 2) are included in this Technical Note. Table 1 contains tabulated results and a description of the model. This twin rotor hover performance, in practical engineering coordinates of $C_{P\ twin}$ versus $C_{T\ twin}$, is shown in Fig. 1. The secondary variable is overlap expressed as a ratio of distance between the shafts, d, to the rotor diameter, D. The test results form visible

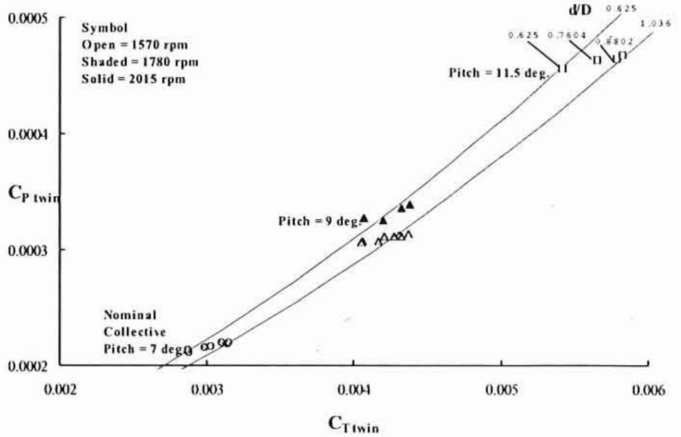

Fig. 1. Stepniewski 1947 co-planar, twin rotor hover experimental results with varying overlap.

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Table 1. Stepniewski 1947 Overlap Test Configuration and Hover Performance Data

Source Part II	Overlap (1-d/D) [%]	Rotor Speed [rpm]	Collective Pitch [deg]	Air Density Ratio	Rotor(s) Total Thrust [lbs]	Rotor(s) Total Power [hp]	Rotor Coefficients (based on $2\pi R^2$ when twin) Thrust C_T	Power C_P
Table I	Fwd. Only	1570	7	0.9697	9.53	0.4039	0.003043	0.0002156
Page 16	Rear Only	1570	7	0.9697	10.47	0.4374	0.003341	0.0002335
	-3.65	1570	7	0.9697	19.72	0.8183	0.003146	0.0002184
	11.98	1570	7	0.9697	19.43	0.8235	0.003101	0.0002198
	23.96	1570	7	0.9697	18.97	0.8106	0.003027	0.0002164
	37.50	1570	7	0.9697	18.07	0.7926	0.002883	0.0002116
Table II	Fwd. Only	1570	9	0.9760	13.53	0.5841	0.004292	0.0003098
Page 17	Rear Only	1570	9	0.9760	14.50	0.6330	0.004598	0.0003358
	-3.65	1570	9	0.9760	27.55	1.1839	0.004368	0.0003140
	11.98	1570	9	0.9760	27.17	1.1787	0.004307	0.0003126
	23.96	1570	9	0.9760	26.55	1.1736	0.004210	0.0003112
	37.50	1570	9	0.9760	25.57	1.1607	0.004054	0.0003078
Table III	Fwd. Only	1570	11.5	0.9760	18.25	0.8415	0.005787	0.0004463
Page 18	Rear Only	1570	11.5	0.9760	20.00	0.9342	0.006342	0.0004955
	-3.65	1570	11.5	0.9760	36.75	1.7605	0.005827	0.0004669
	11.98	1570	11.5	0.9760	36.50	1.7502	0.005787	0.0004642
	23.96	1570	11.5	0.9760	35.65	1.7451	0.005652	0.0004628
	37.50	1570	11.5	0.9760	34.15	1.7193	0.005415	0.0004560
Table IV	Fwd. Only	1780	7	0.9775	12.63	0.6047	0.003111	0.0002197
Page 19	Rear Only	1780	7	0.9775	13.84	0.6613	0.003410	0.0002403
	-3.65	1780	7	0.9775	25.58	1.2122	0.003150	0.0002202
	11.98	1780	7	0.9775	25.22	1.2096	0.003106	0.0002198
	23.96	1780	7	0.9775	24.23	1.1890	0.002984	0.0002160
	37.50	1780	7	0.9775	23.33	1.1736	0.002874	0.0002132
Table V	Fwd. Only	1780	9	0.9736	17.28	0.8544	0.004274	0.0003117
Page 20	Rear Only	1780	9	0.9736	18.47	0.9265	0.004568	0.0003380
	-3.65	1780	9	0.9736	34.95	1.7064	0.004322	0.0003113
	11.98	1780	9	0.9736	34.52	1.7064	0.004268	0.0003113
	23.96	1780	9	0.9736	33.69	1.6859	0.004166	0.0003075
	37.50	1780	9	0.9736	32.78	1.6781	0.004053	0.0003061
Table VI	Fwd. Only	2015	9	0.9773	24.14	1.4194	0.004641	0.0003557
Page 21	Rear Only	2015	9	0.9773	22.15	1.2724	0.004259	0.0003188
	-3.65	2015	9	0.9773	45.58	2.7038	0.004382	0.0003387
	11.98	2015	9	0.9773	44.94	2.6767	0.004320	0.0003353
	23.96	2015	9	0.9773	43.65	2.5955	0.004196	0.0003252
	37.50	2015	9	0.9773	42.27	2.6110	0.004063	0.0003271

Model Configuration

Diameter = 4 ft. Chord = 1.5 inch Blades Per Rotor = 3 Solidity = 0.05968 Twist = 0 deg. Airfoil = NACA 0012
Collective = Varies, Nominally 7, 9 and 11.5 degrees
RPM = 1570, 1780 and 2015 Tip Speed = 329.8, 382.8, and 422.0 fps
Hub to hub distance = 49.75, 42.25, 36.5 and 30 inches
Overlap (1-d/D) = -3.6458, 11.9792, 23.9583 and 37.50 percent

trends in rotor system hover performance with increasing overlap *at constant collective pitch.* Approximate fairings through the data at d/D of 1.0365 and 0.6250 are shown on Fig. 1. (Note that the data set at 2015 RPM and 9 degree collective pitch appears out of line with the lower RPM results; however, the trend with overlap is quite consistent.) Figure 1 clearly shows that overlapping *at constant collective pitch* reduces thrust much more than it reduces power.

Historical Footnote

The first application and extrapolation of this meager (some might say) amount of data was to the Piasecki PV-14 tandem rotor helicopter. This helicopter was initially designated by the U. S. Navy as the XHJP-1, later to become the HUP series in production. The HUP was the first tandem rotor helicopter intermeshed (i.e., overlapped) to a d/D on the order of 0.62. This small, 6000 pound gross weight, piston powered machine, was followed by the CH-46 at 20,000 pounds and then the CH-47 with gross weight now approaching 50,000 pounds. The two, larger, very successful, tandem rotor helicopters had slightly less overlapping (i.e., d/D = 0.65).

In Mr. Stepniewski's original report, Ref. 2, analysis of each data set was presented. The basic experimental trends of thrust (in pounds) and power (in horsepower) versus rotor overlap (1-d/D in percent) were compared to the theory Mr. Stepniewski included as Appendix A to the report submitted to the U. S. Navy. The conclusions of this Part II report noted that the theory and model test results were in agreement and that "Furthermore, in flight tests of the XHJP-1 helicopter, good correlation was found between the predicted and measured performance in hovering and vertical flight." In fact, the model test data was considered "slightly more conservative" than the theory and was "recommended for practical design."

The beauty in the twin rotor experiment Mr. Stepniewski performed lies in *testing for the increment in performance* by sliding the two rotors together while holding all other variables constant. This approach illuminated the 8 to 12 percent overlap influences on performance even though the absolute data is, perhaps, no better than ±3 to ±5 percent in either $C_{P\,twin}$ or $C_{T\,twin}$. There is, of course, some experimental data scatter even with this most fundamental testing approach. However Fig. 1 shows experimental scatter was substantially reduced. Mr. Stepniewski and his small group obtained the major answers to how overlapping affects twin rotor hover performance with just 36 thrust-power data points!

Twin Rotor Hover Performance With Untapered Blades Having $\theta_{(x)} = \theta_{tip}$ / x Twist Distribution

The hover performance of equal solidity, twin rotor systems (having blades that are untapered with a $\theta_{(x)} = \theta_{tip}/x$ pitch angle distribution) is summarized with just a few equations given some fundamental background. The amount of co-planar overlap is defined by the ratio of hub separation distance, d, to rotor diameter, D. This ratio, d/D, dictates the portions of geometric planform area that are either non-overlapped (nov) or overlapped (ov). The simple geometry of this problem is seen with Fig. 2, which outlines the planform view of twin, equal diameter, overlapped rotors. The area of one segment, or its mirror image as Fig. 2 shows, can be found in elementary science handbooks. For twin rotor purposes, this area is defined by d/D as

$$\text{Area Segment 1} = \text{Area Segment 2} = R^2\left[\cos^{-1}\left(\frac{d}{D}\right) - \left(\frac{d}{D}\right)\sqrt{1-\left(\frac{d}{D}\right)^2}\right] \quad (1)$$

Johnson (Ref. 6) usefully defines a segment area parameter, "m" as

$$m \equiv \frac{2}{\pi}\left[\cos^{-1}\left(\frac{d}{D}\right) - \left(\frac{d}{D}\right)\sqrt{1-\left(\frac{d}{D}\right)^2}\right] \quad (2)$$

and thus, the total, non-overlapped, planform projected, geometric area, A_{nov}, of both rotors is

$$A_{nov} = 2\pi R^2 (1 - m) \quad (3)$$

Following similar logic, the total overlapped, planform projected, geometric area, A_{ov}, is

$$A_{ov} = 2\pi R^2 \left(\frac{m}{2}\right) \quad (4)$$

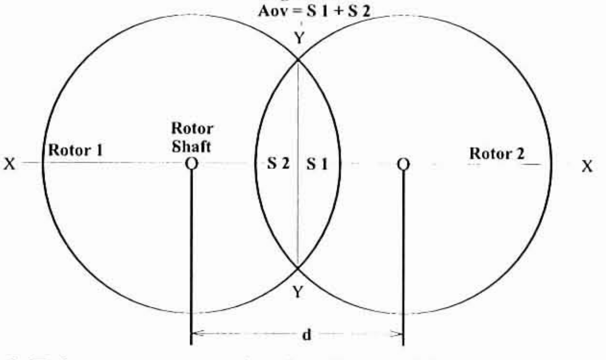

Fig. 2. Twin rotor geometry involves the area of overlapping circles and the basic geometry of segments.

The planform projected, geometric area of a twin rotor configuration, A_{geo} accounting for any amount of overlap defined by d/D (or m) is, of course, simply

$$A_{geo} = 2\pi R^2\left(1 - \frac{m}{2}\right) \quad (5)$$

Note that when the two rotor discs are just touching at their perimeters, d/D = 1 and, m = 0 from Eq. (2). Therefore, $A_{geo} = 2\pi R^2$. At the most overlapped condition of a coaxial twin rotor system, d/D = 0 and m = 1 so that $A_{geo} = \pi R^2$.

Hover thrust and power for co-planar, overlapped configurations, expressed in terms of tip collective pitch, are easily derived. Since the blades are untapered, solidity (using the blade number of one rotor) can be used to scale the tip collective pitch as Knight and Hefner showed in Ref. 7. Therefore, let

$$\Theta_{tip} \equiv \frac{16}{a}\left(\frac{\theta_{tip}}{\sigma}\right) \text{ and } \sigma = \frac{bcR}{\pi R^2} \text{ with both rotors equal geometry} \quad (6)$$

Simplistic airfoil lift and drag aerodynamics for all blade elements are assumed as

$$C_\ell = a\,\alpha \text{ and } C_d = C_{do} + kC_\ell^2 \quad (7)$$

Finally, the induced velocity in the non-overlapped region is identical to the induced velocity of the single, isolated rotor according to blade element-momentum theory. That is, assuming both rotors have identical geometry,

$$\frac{16}{a\sigma}\frac{V_{nov}}{V_t} = \left[\sqrt{1 + 2x\left[\frac{16\theta_{tip}}{a\sigma x}\right]} - 1\right] = \sqrt{1 + 2\Theta_{tip}} - 1 \quad (8)$$

The induced velocity in the overlapped region was derived by Mr. Stepniewski who first published his work in 1948 in Ref. 1. Today the somewhat revised discussion and derivation is contained in Ref. 5. Following this approach, uniform induced velocity will also occur in the overlapped region with constant chord blades having a $\theta_{(x)} = \theta_{tip}$ / x twist distribution. When rotor 1 and rotor 2 have equal geometry, it follows that the magnitude of this uniform induced velocity in the overlapped region will be

$$\frac{8}{a\sigma}\frac{V_{ov}}{V_t} = \sqrt{1 + \frac{1}{2}\left[\theta_{tipR1} + \theta_{tipR2}\right]} - 1 = \sqrt{1 + \Theta_{tip}} - 1 \quad (9)$$

With the above fundamentals in hand, integration of the blade element-momentum equation yields the twin rotor thrust coefficient as

$$C_{T\,twin} \equiv \frac{T_1 + T_2}{\rho(2\pi R^2)V_t^2} = \frac{a^2\sigma^2}{32}\left\{\frac{1}{4}\left(\sqrt{1 + 2\Theta_{tip}} - 1\right)^2(1 - m) + \frac{1}{2}\left(\sqrt{1 + \Theta_{tip}} - 1\right)^2(m)\right\} \quad (10)$$

Twin rotor power coefficient is defined as $C_{Ptwin} \equiv \frac{P_1 + P_2}{\rho(2\pi R^2)V_t^3}$. The total power equals the sum of (a) induced power due to collective pitch, (b) minimum profile power and (c) delta profile power due to collective pitch. The three elements, found by integrating blade element-momentum equations, are

(a)

$$C_{Pi\,twin} = \frac{a^3\sigma^3}{512}\left\{\frac{1}{4}\left(\sqrt{1 + 2\Theta_{tip}} - 1\right)^3(1 - m) + \left(\sqrt{1 + \Theta_{tip}} - 1\right)^3(m)\right\} \quad (11)$$

(b)

$$\text{Min. } C_{Po\,twin} = \frac{\sigma C_{do}}{8} \quad (12)$$

(c)

$$\Delta C_{Po\,twin} = \frac{a^3\sigma^3(ka)}{512}\left\{\left(\Theta_{tip} + 1 - \sqrt{1 + 2\Theta_{tip}}\right)^2(1 - m) + \left(\Theta_{tip} + 2 - 2\sqrt{1 + \Theta_{tip}}\right)^2\left(\frac{m}{2}\right)\right\} \quad (13)$$

Twin rotor hovering performance for rotors using untapered blades having a $\theta_{(x)} = \theta_{tip}/x$ twist distribution can be summarized in practical engineering, C_P versus C_T, form. A useful approximation for practically loaded rotors where $C_{T\,twin}/\sigma^2$ lies between 1 and 2 is

$$\text{for } 0 \leq \frac{d}{D} \leq 1$$

$$C_{P\,twin} \cong \frac{\sigma C_{do}}{8} + \frac{4k}{\sigma} C_{T\,twin}^{2} + \left[\sqrt{2} - \frac{\sqrt{2}}{2}\left(\frac{d}{D}\right) + \left(1 - \frac{\sqrt{2}}{2}\right)\left(\frac{d}{D}\right)^2\right] \frac{C_{T\,twin}^{3/2}}{\sqrt{2}} \tag{14}$$

The third term in Eq. (14) is the minimum induced power of a hovering twin rotor having the assumed blade geometry. If the two rotors are not overlapped, then $d/D \geq 1$ and $m = 0$ in which case the twin rotor induced power reduces to the ideal power of two isolated single rotors. The fully overlapped configuration, a coaxial, has $d/D = 0$ and $m = 1$. The coaxial twin rotor induced power is $\sqrt{2}$ times the induced power of two isolated single rotors.

The preceding simple theory confirms the experimental trends Mr. Stepniewski and his team observed in their 1947 model test. The variation of rotor system thrust and power when twin rotors are overlapped is quite dependent on whether collective pitch is held constant or total thrust is held constant. If collective pitch is held constant and overlap is varied, then induced power decreases slightly at low collective pitch, but increases slightly at high collective pitch. In either case, the primary change is a large reduction in thrust with overlapping as Fig. 1 shows. However, if overlap is accomplished at constant total thrust, induced power always increases. Perhaps surprisingly, delta profile power is virtually independent of overlap (with the assumed blade geometry) and simply follows thrust as Eq. (14) indicates.

Concluding Remarks

The benchmark experimental results that Mr. Stepniewski provided deserve continuous review as succeeding hover performance theories are developed. Mr. Stepniewski noted in Appendix A of the 1948 report to the U. S. Navy that the calculation of thrust and induced power could be obtained using momentum theory and hand integration once the induced velocity distribution was defined. As for calculating profile power, while it was possible, it "would be extremely laborious and time consuming." Mr. Stepniewski offered a very approximate shortcut to obtain the profile power. Since 1948, more computing power has become available, free wake technology has advanced, CFD is proliferating and multi-rotor aircraft are still being developed. However, to this author's knowledge, no tandem rotor hover performance theory with results compared to Mr. Stepniewski's test data has been published. The challenge is there.

Acknowledgment

Mr. Wieslaw Z. Stepniewski participated in every tandem rotor helicopter developed by the now Boeing Helicopters Division. Mr. Stepniewski, better known as just "Steppy," sent me an original of this very valuable report in 1995. I made a copy (returned the original) and put the raw data into a Microsoft EXCEL spread sheet. Steppy answered all my questions about the data acquisition and reduction steps. I was able to reaffirm the results with only minor differences. With Steppy's encouragement and consent, the thrust and power data is included in this technical note. This data, which lies behind the K_{ov} factor, should be of considerable value to those current and future investigators who want to create multirotor free wake technology.

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